

## Problem 1.16

[Difficulty: 3]

**1.16** The English perfected the longbow as a weapon after the Medieval period. In the hands of a skilled archer, the longbow was reputed to be accurate at ranges to 100 m or more. If the maximum altitude of an arrow is less than  $h = 10$  m while traveling to a target 100 m away from the archer, and neglecting air resistance, estimate the speed and angle at which the arrow must leave the bow. Plot the required release speed and angle as a function of height  $h$ .

**Given:** Long bow at range,  $R = 100$  m. Maximum height of arrow is  $h = 10$  m. Neglect air resistance.

**Find:** Estimate of (a) speed, and (b) angle, of arrow leaving the bow.

**Plot:** (a) release speed, and (b) angle, as a function of  $h$

**Solution:** Let  $\vec{V}_0 = u_0 \hat{i} + v_0 \hat{j} = V_0 (\cos \theta_0 \hat{i} + \sin \theta_0 \hat{j})$

$$\Sigma F_y = m \frac{dv}{dt} = -mg, \text{ so } v = v_0 - gt, \text{ and } t_f = 2t_{v=0} = 2v_0/g$$

Also, 
$$mv \frac{dv}{dy} = -mg, \text{ } v dv = -g dy, \text{ } 0 - \frac{v_0^2}{2} = -gh$$

Thus 
$$h = v_0^2 / 2g \quad (1)$$

$$\Sigma F_x = m \frac{du}{dt} = 0, \text{ so } u = u_0 = \text{const}, \text{ and } R = u_0 t_f = \frac{2u_0 v_0}{g} \quad (2)$$

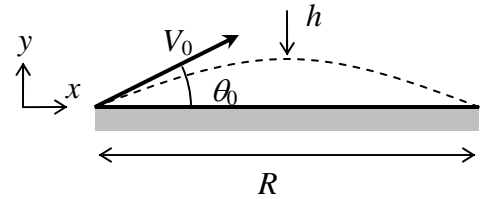
From Eq. 1: 
$$v_0^2 = 2gh \quad (3)$$

From Eq. 2: 
$$u_0 = \frac{gR}{2v_0} = \frac{gR}{2\sqrt{2gh}} \quad \therefore u_0^2 = \frac{gR^2}{8h}$$

Then 
$$V_0^2 = u_0^2 + v_0^2 = \frac{gR^2}{8h} + 2gh \quad \text{and} \quad V_0 = \left( 2gh + \frac{gR^2}{8h} \right)^{\frac{1}{2}} \quad (4)$$

$$V_0 = \left( 2 \times 9.81 \frac{\text{m}}{\text{s}^2} \times 10 \text{ m} + \frac{9.81 \text{ m}}{8 \text{ s}^2} \times 100^2 \text{ m}^2 \times \frac{1}{10 \text{ m}} \right)^{\frac{1}{2}} = 37.7 \frac{\text{m}}{\text{s}}$$

From Eq. 3: 
$$v_0 = \sqrt{2gh} = V_0 \sin \theta, \theta = \sin^{-1} \frac{\sqrt{2gh}}{V_0} \quad (5)$$



$$\theta = \sin^{-1} \left[ \left( 2 \times 9.81 \frac{\text{m}}{\text{s}^2} \times 10 \text{ m} \right)^{\frac{1}{2}} \times \frac{\text{s}}{37.7 \text{ m}} \right] = 21.8^\circ$$

Plots of  $V_0 = V_0(h)$  (Eq. 4) and  $\theta_0 = \theta_0(h)$  (Eq. 5) are presented below:

